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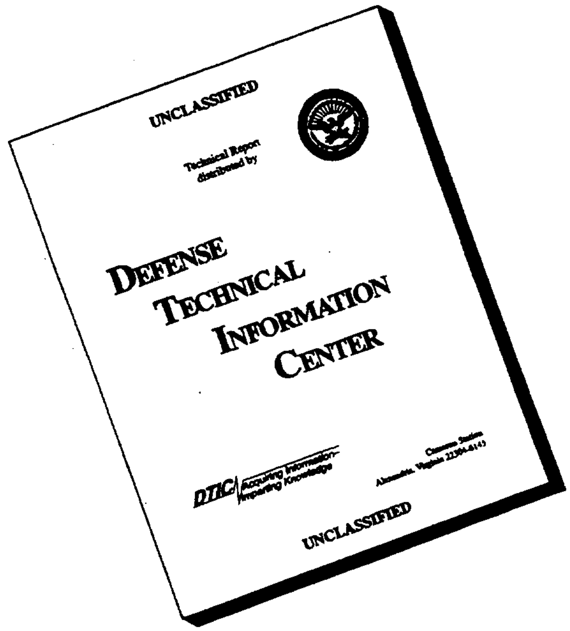


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# FINAL REPORT FOR THE GRANT N00014-89-J-1922

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# Introduction

In a paper presented in 1985 at the Nashville meeting of the Acoustical Society of America we suggested that bubble clouds could oscillate in collective modes that would have a significant effect on underwater sound. For example, bubble clouds form strctures capable of oscillating at frequencies as low as a few tens of Hz, even though the constituent bubbles, in isolation, might have natural frequencies of tens of kHz or more. Similarly, the large contrast between the index of refraction of pure liquid and bubbly liquid gives rise to strong reflection and scattering processes that might well explain the unexpectedly large backscattering data at low grazing angles.

The work carried out under this grant has developed these ideas with particular emphasis on the backscattering from bubble clouds.

We have studied the acoustic excitation of bubble cloud as a whole by adapting tools developed for non-homogeneous media in which the bubbly liquid is treated, in an average sense, as a continuum and the individuality of the bubbles lost. The bubble cloud is modelled as a region filled of a complex fluid having a dispersion relation different from the one prevailing in the surrounding pure water. The Helmholtz equation is solved in the bubble region and in the pure water and the two solutions are matched by suitable boundary conditions. In this way it is possible to simulate noise emission as well as scattering processes. This mathematical model has been very precisely validated by comparison with laboratory experiments conducted with artificial bubble clouds.

The results of the work are documented in a series of papers listed below. The first page of each paper is also reproduced below. Additional information on the work carried out under this grant is provided in the annual summaries submitted to ONR and published in part in the series of annual volumes titled *Ocean Acoustics Program Summary.*

## Student support

The following is a list of the students supported wholly or partially under the ONR grant:

1. ADRIANO M. LEZZI. Doctoral dissertation title: *Topics in Free-Surface Flows.* Johns Hopkins, 1990

2. NAN Q. LU. Doctoral dissertation title: *Bubble Clouds as Sources and Scatterers of Underwater Sound.* Johns Hopkins, 1990

3. XIN CHENG. M.S. in 1992. Worked on analysis of backscattering data from Lake Seneca.

4. TONY BOUQUOT. Judged unsuitable. Left program after one year.

5. KAUSIK SARKAR. Doctoral dissertation title: *Effective Boundary Conditions for Rough Surfaces & Acoustics of Oceanic Bubbles.* Johns Hopkins, March 1994

6. HE YUAN. Doctoral dissertation title: *Dynamics of one and two bubbles in liquids.* Johns Hopkins, April 1996 (expected)

## Papers

The following list does not include oral presentations at meetings of the Acoustical Society of America or at the American Physical Society Fluid Dynamics Division.

1. Atchley, A.A. and Prosperetti, A. The Crevice Model of Bubble Nucleation, *J. Acoust. Soc. Am.*, **86**, 1065-1084, 1989

2. Oğuz, H.N. and Prosperetti, A. A generalization of the Impulse and Virial Theorems, *J. Fluid Mech.*, **218**, 143-162, 1990

3. Yoon,S.W., Crum, L.A., Prosperetti, A. and Lu, N.Q. An Investigation of the Collective Oscillations of a Bubble Cloud, *J. Acoust. Soc. Am.*, **89**, pp. 700-706, 1991

4. Oğuz, H.N., Prosperetti, A., and Lezzi, A. Examples of Air-Entraining Flows, *Phys. Fluids A*, **4**, 649-651, 1992

5. Prosperetti, A., Lu, N.Q., and Kim, H.S. Active and Passive Acoustic Behavior of Bubble Clouds at the Ocean's Surface, *J. Acoust. Soc. Am.* **93**, 3117-3127, 1993

6. Sarkar, K. and Prosperetti, A. Backscattering of Underwater Noise by Bubble Clouds, *J. Acoust. Soc. Am.*, **93**, 3128-3138, 1993.

7. Nicholas, M., Roy, R.A., Crum, L.A., Oğuz, H.N. and Prosperetti, A. Sound Emissions by a Laboratory Bubble Cloud, *J. Acoust. Soc.* **95**, 3171-3182, 1994

8. Oğuz), H.N. and Prosperetti, A. Dynamics of Bubble Growth and Detachment from a Needle, *J. Fluid Mech.* **257**, 111-145, 1993.

9. Sarkar, K. and Prosperetti, A. Coherent and incoherent scattering by oceanic bubbles, *J. Acoust. Soc. Am.* **96**, 332-341, 1994.

10. Prosperetti, A. Bubble mechanics: luminescence, noise, and two-phase flow. In *Theoretical and Applied Mechanics 1992*, Bodner, S.R. et al. eds. Elsevier, 1993, pp. 355-369.

11. Prosperetti, A. Bubble dynamics: Some things we did not know 10 years ago. In *Bubble Dynamics and Interface Phenomena*, Blake, J.R. et al. eds. Kluwer, 1994, pp. 3-16.

12. Prosperetti, A. Linear waves in bubbly liquids. In *Waves in Liquid/Gas and Liquid/Vapour Two-Phase Systems*, Morioka, S. and van Wijngaarden, L. eds. Kluwer, 1995, pp. 55-65.

# The crevice model of bubble nucleation

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The crevice model for heterogeneous nucleation of bubbles in water in response to a decreasing liquid pressure is studied. The model neglects gas-diffusion effects and is therefore more suited for acoustic than for flow cavitation. It is argued that previous work has overlooked the essential requirement of *unstable* growth of the interface in the crevice. As a consequence, the available results are incorrect in some cases. Another feature of the model which is considered is the process by which the interface moves out of the crevice. It is concluded that, depending on circumstances, the conditions for this step may be more stringent than those for the initial expansion of the nucleus inside the crevice. Some numerical examples are given to illustrate the complex behavior of nuclei, depending of geometrical parameters, gas saturation, contact angles, and other quantities.

PACS numbers: 43.35.Ei, 43.30.Nb

## INTRODUCTION

The nature of nucleation and cavitation processes in water has been investigated in the literature since the midnineteenth century.[1–30] A basic conclusion drawn from these studies is that only in rare instances, if at all, does nucleation occur within the bulk of the homogeneous liquid. In order to explain the results of the experiments, it has been necessary to postulate the existence of inhomogeneities in the liquid on which the observed nucleation originated. These inhomogeneities, be they free bubbles, dirt particles, clusters of organic or ionic molecules, or due to a cosmic ray or other form of radiation, have been given the generic name of *cavitation nuclei*. In general (excluding radiation-induced cavitation), cavitation nuclei are long lived and are comprised at least in part by a volume of gas.[16] The first of these characteristics excludes free bubbles from the list of nucleation candidates for cavitation in undisturbed liquids which have been left standing for some time. Free bubbles will quickly dissolve in liquids that are not supersaturated with gas,[31] and bubbles having radii of less than a critical value will dissolve even in a supersaturated liquid.[9,31] Bubbles of larger than critical radius will grow in a supersaturated liquid. In either case, free bubbles are unstable and the liquid will soon be free of them. This instability must be eliminated by any plausible nucleation model. Of the many proposed nucleation models, the two most successful ones are the varying-permeability model[26,27,29] and the crevice model.[6,16,20,25]

The varying-permeability model, which employs a skin of surface-active molecules to stabilize the nucleus, has been applied mainly to bubble formation in supersaturated liquids.[26] The model is plausible and in fact the proposed nuclei have been observed microscopically.[29] The crevice model postulates that small pockets of gas are stabilized at the bottom of cracks or crevices found on hydrophobic solid impurities in the liquid. This model has been applied to various types of cavitation, but its greatest success appears to be in the explanation of acoustic cavitation processes.[16,20,25] It is fair to say that, in the light of present knowledge, there are insufficient elements to decide between these two models, which, in fact, are not mutually exclusive. Both kinds of nuclei may exist in nature with a prevalence of one form or the other, depending on the situation.

The first quantitative application of the crevice model was made by Harvey[6] in a study of bubble formation in animals. Strasberg[16] was the first to apply it to acoustic cavitation and was able to explain the dependence of the acoustic cavitation threshold on the gas content and prepressurization of the liquid. The acoustic cavitation threshold is the pressure amplitude which must be applied to a liquid in order to cause the onset of cavitation. Apfel[20] extended Strasberg's results to include the threshold's dependence on vapor pressure, temperature, and crevice size. Crum[25] further extended the crevice model to include the effect of surface tension on contact angles.

In this paper we reexamine the crevice model and show that the nucleation criterion used by previous investigators is incomplete in that it does not include the essential requirement of mechanically unstable growth of the nucleus as the pressure falls. This condition can lead to substantial differences in some cases, particularly at the higher gas concentrations. We also consider the model in parameter ranges in which it has never been studied before. Finally, we include an analysis of the fate of the nucleus when its surface reaches the crevice mouth. In some cases, the growth past this point may require lower absolute pressures than those necessary for the initial growth. In these cases too the cavitation threshold that we calculate is at variance with that predicted by the previous versions of the crevice model.

From our study it appears that, depending on the parameter values associated with each nucleus, the crevice model can exhibit a bewildering variety of behaviors, which does not seem to have been appreciated before. We believe that many of the earlier conclusions on its physical implica-

  

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# A generalization of the impulse and virial theorems with an application to bubble oscillations

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In the first part of the paper it is shown that the impulse and virial theorems of inviscid incompressible fluid mechanics are special cases of a more general theorem from which an infinity of relations can be obtained. Depending on the problem, only a finite number of these relations may be independent. An application of these results is in the approximate study of the hydrodynamic interaction of bodies. As an example, in the second part of the paper, the case of two freely translating, nonlinearly pulsating bubbles is considered. It is found that in certain parameter ranges the force between the bubbles has a sign opposite to what would be expected on the basis of the linear theory of Bjerknes forces.

## 1. Introduction

Consider $N$ closed material surfaces $S_i$ in a finite or infinite region $\Omega$ occupied by a perfect fluid in irrotational motion. Then Blake & Cerone (1982) proved the following relation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\sum_{i=1}^{N}\int_{S_i}\phi \boldsymbol{n}\,\mathrm{d}S_i = \int_{B}[\tfrac{1}{2}\boldsymbol{u}\cdot\boldsymbol{u}\boldsymbol{n}-(\boldsymbol{n}\cdot\boldsymbol{u})\,\boldsymbol{u}]\,\mathrm{d}S_{\mathrm{B}}. \tag{1.1}$$

In this relation $\phi$ is the velocity potential, $\boldsymbol{u} = \nabla\phi$ is the velocity field, and $\boldsymbol{n}$ is the unit normal directed away from the fluid. By $B$ we denote all the material surfaces bounding the region $\Omega$ other than $S_1, S_2, \ldots$ and the surface at infinity. In the absence of any boundary at a finite distance from the bodies, the right-hand side vanishes and this relation proves the time independence of the sum of integrals in the left-hand side, which is identified with the *impulse* of the fluid (or, more precisely, with the impulse divided by the density). Benjamin & Ellis (1966) and Blake and co-workers (Blake 1983, 1988; Blake & Cerone 1982; Blake & Gibson 1981, 1987; Blake, Taib & Doherty 1986) have given a number of examples of the application of this theorem in bubble dynamics.

Another integral theorem, only valid for an infinite region $\Omega$, has been proven by Benjamin (1987) and, in a different way, by Longuet-Higgins (1989). In the previous notation this theorem may be written

$$\frac{\mathrm{d}}{\mathrm{d}t}\sum_{i=1}^{N}\rho\int_{S_i}-\phi\boldsymbol{x}\cdot\boldsymbol{n}\,\mathrm{d}S_i = -5E_{\mathrm{K}}+\sum_{i=1}^{N}\int_{S_i}(p-p_\infty)\,(\boldsymbol{x}\cdot\boldsymbol{n})\,\mathrm{d}S_i, \tag{1.2}$$

where $E_{\mathrm{K}}$ is the kinetic energy of the fluid, $p$ is the pressure, $\rho$ is the density and $p_\infty$ the ambient pressure. The sum of integrals in the left-hand side is the *virial* of the motion.

In the present paper we shall generalize these results in two directions. First, we

# An investigation of the collective oscillations of a bubble cloud

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It is well known that ocean ambient noise levels in the frequency range from a few hundred hertz to several tens of kilohertz are well correlated with wind speed. A physical mechanism that could account for some of this sound generation is the production of bubble clouds by breaking waves. A simple laboratory study of the sound generated by a column of bubbles is reported here. From measurements of the various characteristics of this column, good evidence is obtained that the bubbles within the column are vibrating in a collective mode of oscillation. Based upon an assumption of collective oscillations, analytical calculations of the predicted frequency of vibration of this column as well as the dependence of this frequency on such parameters as bubble population and column geometry agree closely with the measured values. These results give evidence that the bubble plumes generated by breaking waves can be a strong source of relatively low frequency ( < 1 kHz) ambient noise.

PACS numbers: 43.30.Nb

## INTRODUCTION

The literature contains substantial evidence that oceanic ambient noise is wind-dependent down at least to frequencies of a few hundred hertz.[1-3] The nature of the source of these low-frequency acoustic emissions is, however, still obscure. In view of this seeming correlation with breaking waves, it was suggested independently by Carey[4-6] and Prosperetti[7-9] that the collective oscillations of *bubble clouds* could explain these emissions. It is the purpose of the present paper to present an experimental confirmation of this possibility, and to show that analytical predictions agree closely with the measurements.

It is well known that bubbles abound in the first several meters under the ocean surface. Their origins are varied (impact of splashes, sprays, rain drops, biological activity, capillary waves of limiting form, and other mechanisms), but probably the most significant source of these bubbles is the breaking of surface waves.[10-12] The precise nature of the mechanism of air entrainment in a breaking wave is still poorly understood,[13] but it is plausible that at the moment at which the liquid surface closes on itself to entrap bubbles, its local velocity is nonzero. The bubble created in this way has therefore some initial kinetic energy and, unless it is spherical and with a balanced initial pressure, some initial potential energy as well. Since a bubble may roughly be regarded as a mechanical oscillator, one expects this initial energy to give rise to volume pulsations with associated acoustic emissions. The reason why such a simple picture is unacceptable as an explanation for the observed low-frequency noise is that the natural angular frequency $\omega_0$ of a bubble of radius $a$ is given approximately by

$$\omega_0 = (1/a)(3\gamma p_0/\rho)^{1/2},$$

where $p_0$ is the ambient pressure, $\rho$ is the water density, and $\gamma$ is the ratio of specific heats. With $p_0 = 10^5$ Pa, $\rho = 10^3$ kg/m$^3$, $\gamma = 1.4$, this relation would require a radius of about 7 mm for a frequency $f \doteq \omega_0/2\pi = 500$ Hz, and it is by no means clear—and actually rather unlikely—that such large bubbles are created in significant numbers.

The collective oscillation hypothesis circumvents this problem by appealing to the well-known fact that a system of coupled oscillators (such as the bubbles in the cloud, for which the coupling is provided by mutual hydrodynamic and acoustic interaction) possesses normal modes the frequencies of which can be substantially lower than the natural frequencies of the individual oscillators. An intuitive argument leading to the same conclusion is as follows. It is well known that a mixture of air bubbles and liquid has a sound speed much lower than that of the pure liquid even at gas volume fractions as low as 1%. As a rough approximation, one can therefore consider a bubble cloud as an acoustic medium surrounded by a rigid enclosure, a system which is capable of normal modes of oscillation depending on its linear dimensions. Since the surrounding liquid is not, however, a rigid enclosure, the energy trapped in the cloud will "leak out" and be detectable as acoustic waves away from the bubble cloud. A very simple argument shows that the ratio of the minimum cloud eigenfrequency $\omega_m$ to the natural frequency $\omega_0$ of the constituent bubbles (assumed to be equal) is of the order of[14]

$$\omega_m/\omega_0 \sim 1/\beta^{1/6} N_b^{1/3},$$

where $\beta$ is the gas volume or void fraction in the cloud and $N_b$ is the total number of bubbles. For example, a region with linear dimensions of the order of 10 cm and a void fraction of 1% could contain enough 1 mm radius bubbles to cause a frequency reduction of one order of magnitude.

The theoretical literature on bubble cloud collective-oscillation phenomena has seen several contributions in the last few years.[4-9,14-17] However, so far, no direct experimen-

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# Examples of air-entraining flows

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Four examples of air-entraining flows at the free surface of a liquid are briefly considered: (a) the transient impact of a jet, (b) the application of an excess pressure, (c) two counter-rotating vortices below the surface, and (d) a disturbance on a vortex sheet.

Despite their widespread occurrence (breaking waves, hydraulic jumps, impacting drops, splashes, jets, and others) and importance (underwater noise generation, water treatment, gas–liquid reactors, metal and glass industry) surprisingly little is known about air-entraining flows. It appears that several different mechanisms are at work depending on the conditions, but they are rather poorly understood.[1–3] In the first important paper on air entrainment by jets falling into a pool of the same liquid, Lin and Donnelly[1] showed that, for a laminar jet, bubbles originate from the breakup of an air film that separates the jet liquid from the pool liquid down to a depth of several diameters below the free surface.[4] In the case of a turbulent jet, on the other hand, air entrainment is intermittent and more intimately related to unsteady processes.

In this Letter, we shall consider several examples of transient flows giving rise to air entrainment. While these examples illustrate some physical aspects of the phenomenon, they also present enough of a fluid dynamical interest to warrant consideration in their own right.

In all the cases to be described, the flow is incompressible, inviscid, and potential, possibly with embedded regions of localized vorticity. Surface tension and gravity effects are described by means of the nondimensional Weber and Froude numbers defined by $\mathrm{We} = \rho U^2 L/\sigma$, $\mathrm{Fr} = U^2/gL$, respectively. Here, $\rho$ and $\sigma$ are the liquid density and interfacial tension, $g$ is the acceleration of gravity, and $U$ and $L$ are suitable velocity and length scales. The time scale is given by $L/U$. The numerical method used is essentially that described in Ref. 5 and is based on a boundary-integral approach.

The first example is that of the transient impact of a vertical cylindrical jet on a plane liquid surface. The jet's impact velocity and radius $R$ are taken as the reference velocity and length scales. At the initial instant the jet is just in contact with the plane free surface of the receiving liquid. We show the free surface configuration at successive instants of time in Fig. 1 for $\mathrm{We} = 20.60$ and $\mathrm{Fr} = 68.03$. With an impact velocity of 1 m/sec and the physical properties of water, the corresponding jet radius would be 1.5 mm. No air entrainment is expected for these parameter values in steady conditions. At first, the jet penetrates into the receiving liquid surrounded by a cylindrical shroud of air. This shroud then collapses due mainly to the action of gravity, thus entrapping air. The calculation was stopped just before the occurrence of contact between the facing liquid surfaces.

In the parameter range of Fig. 1, in which the kinetic energy of the jet is relatively large, we find that the thickness of the air shroud is close to the jet's radius, with little dependence on We and Fr. Furthermore, the bottom of the air shroud advances into the receiving liquid with an almost constant velocity essentially independent of We and Fr and very nearly equal to half of the jet's velocity. A simple argument can be given to explain this feature. When the impact velocity is smaller, however, the mechanics is different. The air film is now much less thick with very pronounced waves on the jet's surface and a much smaller amount of air entrained.

A common transient process giving rise to air entrainment—as evidenced by high-speed movies[1,6]—can be described as follows. Occasionally a disturbance occurs on the surface of the falling jet and is convected downward. When it reaches the surface of the pool, it imparts a local impulse that moves the free surface downward much in the same way as the cavity formed by an impacting object. When this depression is sufficiently deep, it cannot fill before it closes off near the free surface. Thus, in a manner analogous to that found in the previous case, one or more bubbles are entrained.

The discussion of the last example considered in this Letter will make clear that the numerical simulation of this process is very complex. Therefore we look here at a related, but much simpler, situation. We consider a liquid of infinite depth unbounded on one side and bounded by a vertical wall on the other. At time zero, a constant excess pressure $\Delta p \equiv \rho U^2$ begins to act on the liquid surface over a portion of width $L$ adjacent to the wall. Figure 2 (in which the physical system is rotated clockwise by 90°) shows a series of snapshots of the free surface for the case $\mathrm{We} = 6.62$, $\mathrm{Fr} = 50$. For water, these numbers correspond to $L = 1$ mm, $U = 0.7$ m/sec. A consideration of the Bernoulli integral suggests that the applied $\Delta p$ will eventually be overpowered by the hydrostatic head so that the deepening of the cavity will stop. However, the present situation presents obvious similarities with the previous one and a similar sequence of events can therefore be expected.

This problem is somewhat analogous to the steady separated flow past a plate normal to the incoming stream which, in two dimensions and in the absence of gravity and surface tension, admits of an exact closed-form solution. A numerical treatment of the axisymmetric case has been given by Brennen.[7] The analytic solution shows that, in contrast with the previous axisymmetric situation, the

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# Active and passive acoustic behavior of bubble clouds at the ocean's surface

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The emission and scattering of sound from bubble clouds is studied theoretically. It is shown that clouds having a size and air content similar to what might be expected as a consequence of the breaking of ocean waves can oscillate at frequencies as low as 100 Hz and below. Thus cloud oscillations may furnish an explanation of the substantial amount of low-frequency wind-dependent oceanic ambient noise observed experimentally. Detailed results for the backscattering from bubble clouds—particularly at low grazing angles—are also presented and shown to be largely compatible with oceanic data. Although the cloud model used here is idealized (a uniform hemispherical cloud under a plane water free-surface), it is shown that the results are relatively robust in terms of bubble size, distribution, and total air content. A similar insensitivity to cloud shape is found in a companion paper [Sarkar and Prosperetti, J. Acoust. Soc. Am. 93, 3128–3138 (1993)].

PACS numbers: 43.30.Nb, 43.30.Ft, 43.30.Lz

## INTRODUCTION

Recent research has brought to the fore the importance of bubble clouds at the ocean's surface for the propagation and generation of underwater sound (Carey and Bradley, 1985; Carey and Browning, 1988; Prosperetti, 1985, 1988a,b; Lu *et al.*, 1990; Yoon *et al.*, 1991; Lu and Prosperetti, 1993). On the one hand, it has been realized that the clouds can oscillate in collective modes and give rise to acoustic emissions as low as a few tens of Hz irrespective of the size of the constituent bubbles. It is possible that in this fact an explanation may be found of the low-frequency component of wind-dependent oceanic ambient noise that has long puzzled investigators (Wenz, 1962; Kerman, 1984; Carey and Wagstaff, 1986; Kewley *et al.*, 1990). Secondly, several analyses have shown that the backscattering produced by bubble clouds can be quite substantial and preliminary estimates indicate that these entities may be responsible for the unexpectedly high backscattering strengths found experimentally.

The presence of bubbles in substantial numbers in the uppermost several meters of the ocean surface is so well known that a few references will be sufficient in this respect (Monahan, 1971; Thorpe, 1982; Farmer and Vagle, 1989). One may distinguish between the regions of "fresh" bubbles, newly formed by breaking waves, and the "old" bubbles, that are stabilized by still unclear mechanisms and survive long after their formation. Typically, "fresh" bubbles are relatively highly concentrated in clouds very close to the surface, while "old" bubbles are much less dense and are transported downward by turbulence and Langmuir circulation. We refer to these latter agglomeration as bubble "plumes" reserving the term "cloud" for the former.

In this paper we consider both the active and passive facets of the acoustic behavior of bubble clouds. Our model is geometrically simple and consists of hemispherical clouds at the surface of a plane ocean. However, we use a relatively complete model for the bubbly region that goes beyond previous calculations and we solve exactly—rather than in an approximate fashion—the emission and scattering problems. In our basic model the bubble cloud has a uniform spatial distribution of gas and bubble radii. Later, we consider generalizations of this model and try to derive conclusions of more general validity. The dependence of the results on the cloud shape is clearly also important, and is addressed in a separate publication (Sarkar and Prosperetti, 1993a).

Crowther (1980), McDaniel and Gorman (1982, 1983), and McDaniel (1987) have applied incoherent scattering methods to the calculation of backscattering from bubble layers. More recently, McDonald (1991) and Henyey (1991) have treated the backscattering from bubble plumes by the Born approximation. The methods used by these authors are not suitable for the denser bubble assemblies studied in this paper. A detailed comparison of these approaches can be found in Sarkar and Prosperetti (1993a, 1993b).

Perhaps the major piece of information missing for a quantitative description of bubble clouds is the gas concentration by volume, also called void fraction in the multiphase flow literature. Some estimates of this quantity during the active breaking process are as high as 30% (Longuet-Higgins and Turner, 1974; Melville *et al.*, 1992), but this situation of extremely large void fraction must be

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# Backscattering of underwater noise by bubble clouds

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This paper is a continuation of an earlier one [Prosperetti *et al.*, J. Acoust. Soc. Am. **93**, 3117–3127 (1993)] in which the low-frequency backscattering of sound by hemispherical bubble clouds at the ocean's surface was studied. Here, clouds of various geometrical shapes (spheroids, spherical segments, cones, cylinders, ellipsoids) are considered and results in substantial agreement with the earlier ones and with the experiments of Chapman and Harris [J. Acoust. Soc. Am. **34**, 1592–1597 (1962)] are found. The implication is that the backscattering levels are not strongly dependent on the shape of the clouds, which strengthens the earlier conclusion that bubble clouds produced by breaking waves can very well be responsible for the unexpectedly high backscattering levels observed experimentally. The accuracy of the Born approximation used by others for similar problems is also examined in the light of the exact results. Significant differences are found for gas concentrations by volume of the order of 0.01% or higher. Finally, shallow nonaxisymmetric plumes are briefly considered.

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## INTRODUCTION

Recently, the study of the natural mechanisms of underwater noise generation and the anomalously high backscattering levels often encountered in underwater sound propagation have stimulated an intense interest in the physics and properties of oceanic bubble clouds (Carey and Bradley, 1985; Prosperetti, 1985, 1988a, 1988b; Carey and Browning, 1988; Lu *et al.*, 1990; Yoon *et al.*, 1991; McDonald, 1991; Henyey, 1991; Lu and Prosperetti, 1993; Prosperetti *et al.*, 1993). The present paper devoted to the backscattering from clouds of various shapes is a continuation of our research activity in this area. In earlier studies we have considered bubbly layers (Lu and Prosperetti, 1993) and hemispherical clouds (Yoon *et al.*, 1991; Prosperetti *et al.*, 1993). These papers may be consulted for additional references and a more detailed discussion of the motivation and implications of this research. Here we merely stress that the mathematical model that we use for the description of the acoustic properties of bubble clouds has been found to agree remarkably well with experiment (Commander and Prosperetti, 1989; Lu *et al.*, 1990; Yoon *et al.*, 1991; Nicholas *et al.*, 1993), so that it can be used with considerable confidence in the present application.

The main result of our earlier work devoted to hemispherical bubble clouds was that the experimental backscattering data obtained by Chapman and Harris (1962) could be closely reproduced by making reasonable assumptions on the clouds' size and gas content. The purpose of the present study is to strengthen the earlier conclusions by demonstrating the relative insensitivity of those results to the detailed cloud shape. We consider several examples of axisymmetric (spheroidal and other) and nonaxisymmetric bubble clouds and hemicylindrical clouds and again find backscattering levels quite comparable to the experimental ones.

We also address the accuracy of the Born approximation in estimating scattering from bubble clouds. This part of the work is motivated by the use of that approximation in some recent studies (McDonald, 1991; Henyey, 1991). We conclude that the approximation is quite useful up to gas volume fractions of the order of $10^{-2}\%$. In the next to the last section we present an approximate treatment of nonaxisymmetric shallow clouds.

Other theoretical studies of the effect of subsurface bubbles on surface backscattering are available (see, e.g., Crowther, 1980; McDaniel and Gorman, 1982; McDaniel, 1988). However these are focused on higher frequencies and employed a theoretical formulation only suitable for exceedingly small bubble concentrations. An analysis of the relationship between those formulations and the present one is presented in a separate study (Sarkar and Prosperetti, 1993).

## I. MATHEMATICAL MODEL

The mathematical model has been discussed in detail in several preceding papers (see, e.g., Commander and Prosperetti, 1989; Lu and Prosperetti, 1993; Yoon *et al.*, 1991), and so will not be repeated here. It has been shown in those papers that the governing equation for pressure perturbations in the bubbly liquid, dependent on time proportionally to exp $i\omega t$, is a scalar Helmholtz equation with wave number $\kappa$ given by

$$\kappa^2=k^2+\frac{4\pi\omega^2 an}{\omega_0^2-\omega^2+2ib\omega}. \tag{1}$$

Here, $k=\omega/c$ (with $c$ the speed of sound in the pure liquid), $a$ is the equilibrium radius of the bubbles, $n$ is the bubble number density, $b$ is the frequency-dependent "damping constant", and $\omega_0$ is the natural frequency of the bubble. Explicit expressions for these quantities can be found in the papers cited before. The gas volume fraction is given by

$$\beta=\tfrac{4}{3}\pi n a^3. \tag{2}$$

  

# Sound emissions by a laboratory bubble cloud

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This paper presents the results obtained from a detailed study of the sound field within and around a cylindrical column of bubbles generated at the center of an experimental water tank. The bubbles were produced by forcing air through a circular array of hypodermic needles. As they separated from the needles the "birthing wails" produced were found to excite the column into normal modes of oscillation whose spatial pressure-amplitude distribution could be tracked in the vertical and horizontal directions. The frequencies of vibration were predicted from theoretical calculations based on a collective oscillation model and showed close agreement with the experimentally measured values. On the basis of a model of the column excitation, absolute sound levels were analytically calculated with results again in agreement with the measured values. These findings provide considerable new evidence to support the notion that bubble plumes can be a major source of underwater sound around frequencies of a few hundred hertz.

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## INTRODUCTION

Wave breaking is the most significant air-entrainment process occurring at the surface of the ocean and gives rise to a large number of bubble clouds (Thorpe, 1982, 1986; Thorpe and Hall, 1983; Monahan and Mac Niocaill, 1986; Monahan and Lu, 1990). It is known that the number and intensity of breaking waves are strongly dependent on the wind speed above the ocean surface (see, e.g., Toba and Koga, 1986; Phillips, 1988; Wu, 1988), and it has also been shown that a correlation exists between wind speed and the intensity of low-frequency (below 1 kHz) ambient sound in the ocean (Wenz, 1962; Piggott, 1964; Perrone, 1969; Kerman, 1984; Kuperman and Ferla, 1985; Wille and Geyer, 1985; Carey and Wagstaff, 1986; Kennedy and Goodnow, 1990; Kewley *et al.*, 1990; Kennedy, 1992). These circumstances have led to the consideration of processes by which breaking waves may produce such low-frequency noise (Wilson, 1980; Kerman, 1984). Carey and co-workers (Carey and Bradley, 1985; Carey and Browning, 1988) and, independently, Prosperetti (1985, 1988a, 1988b) have suggested that collective oscillations of the bubble clouds produced by breaking waves could be responsible for the low-frequency emissions. The argument was essentially that, since the bubbles in the cloud constitute a collection of coupled oscillators, one would expect the existence of normal modes of oscillation of the cloud itself at frequencies far lower than the frequency of oscillation of the individual bubbles. This idea has been explored in, and supported by, a number of subsequent publications (Lu *et al.*, 1990; Yoon *et al.*, 1991; Carey *et al.*, 1993; Prosperetti *et al.*, 1993; Koller and Shankar, 1993; Oğuz, 1994).

In view of the difficulty in gathering oceanic field data on the role of bubble clouds in low-frequency sound generation, the conclusions mentioned above rest mainly on theoretical analyses only partially verified in laboratory experiments. It is therefore important to validate further the theoretical models used so as to gain confidence in their predictions. Initial experiments (Yoon *et al.*, 1991) have shown that bubble clouds are capable of collective oscillations at frequencies far below those of the individual constituent bubbles in excellent agreement with theory. In this paper that work is extended in two significant ways. In the first place, the use of a much more extensive data set renders the measurement of the higher mode frequencies possible with a very good match with theory. Second, the model is extended to the prediction of the *absolute acoustic levels*, again in good agreement with data. This is a very nontrivial point as it presupposes a quantitative understanding of the mechanism by which the bubble cloud is excited. Our conclusion is that the energy imparted to the individual bubbles upon their formation coupled with the spectral width of single-bubble free oscillations accounts for the level of acoustic radiation observed in the experiment. In a separate study (Oğuz, 1994), it is shown that, on the same basis, good predictions of low-frequency oceanic ambient noise can be found. These successful estimates of levels are a stringent test of the theory as it is well known that, in general, it is much easier to match frequencies than levels.

## I. EXPERIMENTAL PROCEDURE

The experimental arrangement used in this work is very similar to the one described in Yoon *et al.* (1991) and

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# Dynamics of bubble growth and detachment from a needle

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Several aspects of the growth and departure of bubbles from a submerged needle are considered. A simple model shows the existence of two different growth regimes according to whether the gas flow rate into the bubble is smaller or greater than a critical value. These conclusions are refined by means of a boundary-integral potential-flow calculation that gives results in remarkable agreement with experiment. It is shown that bubbles growing in a liquid flowing parallel to the needle may detach with a considerably smaller radius than in a quiescent liquid. The study also demonstrates the critical role played by the gas flow resistance in the needle. A considerable control on the rate and size of bubble production can be achieved by a careful consideration of this parameter. The effect is particularly noticeable in the case of small bubbles, which are the most difficult ones to produce in practice.

## 1. Introduction

The production of bubbles by submerged needles or orifices occurs in a large number of technical applications including water treatment, metallurgy, and a variety of chemical plants such as perforated plate columns. Other important applications occur in medicine where blood oxygenators are often based on the same principle (Sutherland, Derek & Gordon 1988; Kurusz 1984; Matkovitch 1984). The process has also some connection and resemblance to bubble departure in boiling, which is a major aspect of industrial operations in the power industry and many others (see e.g. Sluyter *et al.* 1991; Wang & Bankoff 1991 and references quoted therein). In laboratory research on multiphase flow and acoustics, it is often necessary to generate small bubbles in a controlled fashion, which sometimes is found to be a maddeningly difficult task. Finally, one might hope that by studying this process some insight could be gained on bubble formation in air-entraining flows, which are themselves of major importance in many areas of technological as well as of environmental concern.

The practical importance of this process and its mechanics, unexpectedly rich in subtleties, have motivated a considerable number of studies (Clift, Grace & Weber 1978; Tsuge & Hibino 1983). A detailed review of the literature up to about 1970 is given by Kumar & Kuloor (1970); this article contains a wealth of experimental facts and data and describes several theoretical models based on force balances expressed in terms of numerically or exactly solvable ordinary differential equations. In all of these simplified models the bubble is taken to grow spherically and the objective is to calculate the volume at its detachment from the needle. It is however known that bubbles are not spherical when they grow, a fact that is crucial for the understanding of the acoustic emissions upon their departure from the needle. Furthermore, methods have been proposed to produce small bubbles by immersing the needle in a flow (Chuang &

# Coherent and incoherent scattering by oceanic bubbles

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A substantial amount of research on acoustic scattering by underwater bubbles is based on the theory of incoherent scattering. More recent work, devoted to much denser bubble assemblies, has instead used effective-media formulations that presuppose coherent effects. Here the mutual relationship between the two approaches is elucidated. It is shown that, underlying the incoherent results, is a WKB approximate solution of the effective equations. As an application, the scattering by tenuous subsurface bubble layers and acoustical bubble counting techniques are examined. Significant differences with previous results are found.

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## INTRODUCTION

Bubble clouds have recently become of great interest to the underwater acoustics community due to their possible role in the production and scattering of low-frequency (from a fraction of a kHz to several kHz) noise (Carey and Bradley, 1985; Carey and Browning, 1988; Prosperetti *et al.*, 1993; Sarkar and Prosperetti, 1993). The mechanism by which these effects are produced involves the acoustic excitation of the cloud as a whole rather than of the individual constituent bubbles. Accordingly, the theoretical tools adapted for the analysis have been found in the theory of nonhomogeneous media in which the bubbly liquid is treated, in an average sense, as a continuum and the individuality of the bubbles lost.

While this theory has been very successfully compared with experiment (Commander and Prosperetti, 1989; Yoon *et al.*, 1991; Nicholas *et al.*, 1994), it represents a departure from the way in which oceanic bubble phenomena have usually been analyzed in the past, when the emphasis was on higher frequencies (tens of kHz and above) and sparser bubble assemblies. In that case the bubbles were treated as individual scatterers or emitters of sound and the relationship between the two points of view is not apparent. It is the purpose of the present paper to clarify this relationship. In so doing, the assumptions underlying the previous approaches will also be clarified and their limitations explored in the light of a more accurate theory.

In part, the differences between the two approaches must be attributed to the differences between coherent and incoherent scattering and emission. However, there are additional elements that make the comparison more interesting.

Although sound emission by individual bubbles is mentioned among the possible oceanic noise sources, little quantitative effort has been devoted to this aspect of bubble activity. Mostly, the previous studies have focused on the acoustic scattering properties of bubbles, and it is to these that the present analysis will be limited. The problems that we study are the high-frequency reverberation and backscattering due to bubbles, and an acoustical technique of bubble counting.

## I. COHERENT AND INCOHERENT SCATTERING

In this paper we shall only be concerned with random arrangements of scatterers. In this case, the basic physics underlying the coherent and incoherent interaction of waves with scatterers is well known (see e.g., Morse and Feshbach, 1953, p. 1494; Carey and Roy, 1993). Incoherent behavior dominates when the scatterers are separated by distances comparable with the wavelength or larger, and leads to a scattered intensity proportional to the number of scatterers $N$. When the scatterers are closely spaced, on the other hand, one observes coherent behavior with a much stronger scattering strength proportional to $N^2$. The transition between the two regimes is continuous: any random assembly of scatterers, however compact in its spatial extension, will give rise to an incoherent component which becomes (relatively) stronger and stronger as the typical acoustic wavelength decreases. Eventually, the incoherent component becomes dominant and the coherent field becomes small, although its vanishing is asymptotic rather than abrupt.

An alternative—but equivalent—description of the situation can be given in terms of ensemble averages. Imagine an ensemble of repeated scattering experiments in which the average conditions are nominally identical, and the only difference lies in the details of the spatial arrangemement of the individual scatterers. Upon averaging the results (e.g., the spectra of the scattered signal) of all the experiments, one would obtain the coherent field, while the difference of the measured field in each experiment from the coherent average would correspond to the incoherent field for each particular experiment. The ensemble average of these incoherent fields vanishes by definition, but the ensemble average of the squares does not and this quantity is proportional to the incoherent scattering intensity.

The equivalence of the two descriptions stems essentially from the equivalence between volume and ensemble averaging for spatially homogeneous systems.

In the situations of concern in underwater acoustics the density of bubbles is generally small, with gas volume fractions ranging from perhaps $10^{-6}$ to a few percent. However, in view of the large compressibility and energy dissipation of bubbles, exceedingly strong effects on sound propagation take place near the upper end of this range.

## II. FOLDY'S THEORY

In a pioneering paper published in 1945, Foldy developed a consistent theory of coherent and incoherent scatter-

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# Bubble mechanics: luminescence, noise, and two-phase flow

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## Abstract

A descriptive treatment of light emission from pulsating bubbles (sonoluminescence), sound generation by rain falling on water and breaking waves, and the propagation of shock waves in bubbly liquids is presented. The first section contains a brief survey of some occurrences of bubbles in literature and in the figurative arts. Considerations on the etymology of the word are presented in the appendix.

## 1. INTRODUCTION

*Πομφόλυξ ο άνθροπος* – Man is a bubble – is an ancient Greek proverb that enjoyed a singular favor in the western culture as an expression of the caducity and impermanence of human life. It is used by a number of Latin and Greek authors such as Varro who, apologizing in the Preface to his *De Re Rustica* (36 b.c.) that the work is not as polished as he would like, says that nevertheless, being 80 years old, he should go ahead with its publication since, "ut dicitur, si est homo bulla, eo magis senex" ("if, as they say, man is a bubble, all the more so is an old man"). Petronius (1st century a.d.), in a mocking passage of the *Satyricon* in which he compares man to inflated walking bags and flies, also says "nos non pluris sumus quam bullis" ("we are no more than bubbles," 42, 4). Lucian (117-180) elaborates: "I've thought of a simile to describe human life as a whole ... You know the bubbles that rise to the surface below a waterfall – those little pockets of air that combine to produce foam? ... Well, that's what human beings are like. They're more or less inflated pockets of air ... but sooner or later they're all bound to go pop" (*Charon* 19).

The advent of Christianity, with its message of hope and salvation, rendered the idea less relevant and probably the only pre-Renaissance textual reference to it is in the *Lexicon* by the 10th-11th century Byzantine scholar Suida ("as a bubble immediately disappears when it is broken, so does the memory of the splendid and powerful upon their death").

However, in the northern post-reformation cultural climate of the 16th and 17th century, the metaphor regained its appeal. Most succint – and first – is Erasmus: "Homo bulla" [man (is a) bubble, *Adagia*, 1508, n. 1990]. Taverner, in his *Proverbs or adagies ... gathered out of the Chiliades of Erasmus* (1539) echoes the concept, and so does sir Thomas Elyot (1545), Golding ("When man seemeth to bee at his best, he is al-

Bubble dynamics:

# Some things we did not know 10 years ago

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**Abstract.** This paper contains a brief review of the author's work on several problem related to bubble dynamics. A "state-of-the-art" model for spherical dynamics is formulated first. On the basis of this model several features of the thermo-fluid mechanical behavior of gas bubbles are then discussed and applications to sonochemistry and mass transfer described. The paper ends with a brief discussion of pressure wave propagation in bubbly liquid and the role played by bubbles in the generation of oceanic ambient noise.

**Key words:** Bubbles, Bubbly liquids, Acoustic cavitation, Oceanic noise

## 1 Introduction

The period since the 1981 IUTAM Symposium on Mechanics and Physics of Bubbles in Liquids [1] held at the California Institute of Technology in Pasadena has seen a tremendous growth in bubble research, both fundamental and applied. I shall make no attempt to review all this new material, both in view of its amount and of the fact that most of its originators are contributors to this volume. Rather, I shall focus on some of the results that my colleagues and I have obtained in the last decade.

I am saddened by the fact that Prof. Milton S. Plesset, the gracious host of the Pasadena meeting, is no longer with us. With him we have lost one of the propulsive forces of our community, a scientist whose contributions have shaped bubble research in our time. Personally, I am even more struck as I sorely miss his enlightening mentorship and guiding voice.

## 2 Spherical Bubble Dynamics

Shortly after the Pasadena meeting Prof. Larry Crum got me involved in his work on bubble levitation [2]. In the attempt to reconcile the data with theory we met some significant differences that, I am sorry to say, have not been resolved to this date. Around the same time I had also started to work on a book on bubble dynamics. Both factors prompted me to look more deeply into the theoretical framework, and especially at the incorporation of liquid compressibility into the well-known Rayleigh-Plesset equation, and at the description of the gas thermo-fluid mechanics. As it turned out, answers to my questions already existed, but were not widely appreciated at the time.

As for liquid compressibility, the problem was not the absence of equations



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# LINEAR WAVES IN BUBBLY LIQUIDS

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Abstract. A summary of the current status of the modeling of the propagation of pressure waves in bubbly liquid is presented. An apparent failure of the theory near and above the resonance frequency of the bubbles is found by comparison with experimental data. In the second part of the paper applications to oceanic noise, laboratory bubble clouds, and resonance-parametric generation of low frequency underwater sound are described.

Key words: Bubbly liquids, Two-phase flow, Oceanic noise

## 1 Foldy's theory

The lowest-order theory of wave propagation in bubbly liquid was essentially worked out by Foldy in a well-known 1945 paper [3], and later applied by him and Carstensen to the analysis of data [4]. The following, although by no means rigorous, is a very simple derivation of Foldy's result.

In the context of potential flow theory, to leading order, bubbles behave as monopoles with strength given by $\dot{v}$, the time derivative of their volume. Consider then $N$ bubbles immersed in an "incident" flow given by a potential $\phi_\infty$. The total flow due to the incident flow plus the effect of the bubbles has then the potential

$$\phi = \phi_\infty + \sum_{j=1}^{N} \frac{\dot{v}_j}{4\pi|\mathbf{x} - \mathbf{x}_j|}, \tag{1}$$

where $\mathbf{x}_j$ is the position of the $j$-th bubble. If $N$ is large, the preceding relation may be approximated as

$$\phi \simeq \phi_\infty + \int \frac{\dot{v}}{4\pi|\mathbf{x} - \mathbf{x}'|} n(\mathbf{x}')\, d^3x', \tag{2}$$

where $n$ is the bubble number density. Upon taking the Laplacian of this expression, since $\phi_\infty$ is regular in the region of interest, we have

$$\nabla^2\phi = n\dot{v}, \tag{3}$$

which is essentially Foldy's result for an incompressible liquid. This can be written in a somewhat different form by noting that, for the linear problem,

$$\mathbf{u} = \nabla\phi \qquad P = -\rho\frac{\partial\phi}{\partial t}, \tag{4}$$



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